A poro-elastic solution for transient fluid flow into a well

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ABSTRACT

The paper presents an analytical solution for a transient two-dimensional radial flow of a compressible fluid into a line well. The problem is formulated in a context of poro-elasticity and the solution fully accounts for effects of stress redistribution around the well as well as the back effects of stress changes on fluid flow. Poro-elastic, fluid-saturated reservoir is considered to be plane and surrounded by an impermeable elastic material of an infinite extent. The governing equation for fluid pressure is derived by considering a general axi-symmetric solution of the theory of elasticity and using compatibility of displacement and stresses at the interface between the reservoir and the surrounding material. The resulting second order integro-differential equation is solved using Hankel transform. In limiting cases of infinitely stiff and infinitely soft host material the governing equation reduces to the standard diffusion equation. Implications of the solution for well testing and reservoir simulation in petroleum engineering are discussed.

Introduction

The production of oil or water from underground reservoirs or injection of displacing fluids results in local changes of the stress field as a consequence of pressure changes in permeable formations surrounding wells. The resulting reservoir deformations tend to alter porosity of the reservoir rock and can have a pronounced effect on conditions of fluid flow.

The theory of poro-elasticity pioneered by Biot (1941) can be used to describe a coupled process of fluid flow and associated stress changes in the host material. Most practical applications, however, such as well testing in petroleum engineering or reservoir simulation, are based on solutions of uncoupled flow equations obtained by neglecting total stress changes in the reservoir. For some problems, such as the assessment ground surface subsidence, the problem of the theory of elasticity is solved separately based on prescribed pressure changes, e.g. Geertsma (1973). Entov and Malachova (1974) give a detailed uncoupled solution for stress changes around a well assuming pressure to follow the solution of the standard transient well equation. They also express an opinion that the back effect of stress change on fluid flow is in most cases very small. On the other hand, a numerical assessment of the effect of global stress changes on oil production from Ekofisk reservoir suggests that such effects are far from negligeable, Sulak et. al., 1991.

The objective of the present paper is to pose and solve a fully coupled poro-elastic problem of transient compressible flow into a line well. A single phase fluid flow in the poro-elastic unbounded plane reservoir is assumed to be radial and two-dimensional. The host rock surrounding the reservoir is treated as impermeable and linearly elastic. Deformations of the reservoir are assumed to be vertical and the influence of the ground surface is neglected. In practical terms, the solution is applicable for flow times such that the radius of a zone affected by pressure changes is small compared to the reservoir depth.

In the conventional treatment of this problem it is commonly assumed that overburden has no stiffness and the vertical stress at the reservoir plane is unaffected by pressure changes in the reservoir. In this case reservoir compaction is completely determined by local pressure changes and the governing equation for transient flow is the well-known parabolic diffusion equation. When the stiffness of the host material is taken into account, local pressure changes create only a potential for compaction. Vertical contraction of the reservoir would tend to induce tensile deformations in the host material creating a reaction that would resist compaction. In the limiting case when the host material is infinitely stiff, reservoir deformations will not take place at all, no matter what pressure change is induced in the reservoir. This case is also described by the conventional well equation assuming that the reservoir material is incompressible. In all other cases, as it will be shown below, the degree of the reservoir compaction is strongly influenced by the relative stiffness of the overburden with respect to the reservoir stiffness.

The interaction of reservoir and overburden is such that pressure change in one location leads to deformations and stress changes all over the reservoir. This, in its turn, affects pressure changes at all reservoir locations. This non-local nature of the reservoir-overburden interaction leads to an integro-differential governing equation for transient flow. This equation is derived and solved below.

Mass Balance in Reservoir

Compressibility of the reservoir affects only the storage term in the transient flow equation. If q is the mass flux of all flowing components, their accumulation (in terms of mass) in a unit volume per unit time is $-divq$, where q is the flow vector. This extra mass has to be accommodated within the pore volume, either by fluid compression/expansion or by changes in the volume of the pore space. If δv_b is an infinitesimal bulk volume through which flow occurs, and $\delta v_p = \phi\delta v_b$ is the pore volume (ϕ - porosity), the continuity of flow can be expressed as follows:

$$-divq = \frac{1}{\delta v_b}\frac{\partial}{\partial t}(\rho\delta v_p) = \phi\frac{\partial\rho}{\partial t} + \rho\frac{1}{\delta v_b}\frac{\partial\delta v_p}{\partial t}\ , \qquad (1)$$

where ρ is the average density of fluids. Its rate of change depends on changes in partial pressures of different components. The second term above accounts for pressure and overburden stress-related changes in pore space. It should be noted that it would be incorrect to write the last term of (1) as $\rho\partial\phi/\partial t$ introducing δv_b under the sign of the time derivative. This is because δv_b changes with time as a result of stress changes caused flow.

Reservoir Material Model

In the subsequent formulation the reservoir material will be treated as poro-elastic. This implies that variation in pore pressure and external confinement results in changes of both pore and bulk volumes. Assuming that the pore volume v_p and the bulk volume v_b are functions of pore pressure p and hydrostatic stress σ, i.e. $v_p(p, \sigma)$ and $v_b(p, \sigma)$, the incremental volumetric response of infinitezinal volumes δv_b and δv_p can be expressed as follows:

$$\delta \dot{v}_p = \frac{\partial \delta v_p}{\partial p}\dot{p} + \frac{\partial \delta v_p}{\partial \sigma}\dot{\sigma} = \delta v_p C_{pp}\dot{p} - \delta v_p C_{p\sigma}\dot{\sigma} \tag{2}$$

$$\delta \dot{v}_b = \frac{\partial \delta v_b}{\partial p}\dot{p} + \frac{\partial \delta v_b}{\partial \sigma}\dot{\sigma} = \delta v_b C_{bp}\dot{p} - \delta v_b C_{b\sigma}\dot{\sigma} \tag{3}$$

where compressibilities $C_{pp}, C_{p\sigma}, C_{bp}, C_{b\sigma}$ are positive and defined through partial derivatives of respective volumes. Note that stress σ is considered positive when compressive.

Physical arguments put forward by Zimmerman et. al., 1986, suggest that only two out of four compressibilities are independent. In the subsequent text the reservoir material will be described in terms of bulk compressibility $C_r = C_{b\sigma}$ and compressibility of solid matrix, C_m. Other compressibilities are expressed in terms of C_r, C_m as follows, (Zimmerman et. al., 1986)

$$C_{bp} = C_r - C_m \tag{4}$$

$$C_{p\sigma} = (C_r - C_m)/\phi \tag{5}$$

$$C_{pp} = [C_r - (1 + \phi)C_m]/\phi \tag{6}$$

In order to reduce the number of dimensional constants it is convenient to use ratio of compressibilities $\alpha = C_m / C_r$. This ratio is small and will be shown to have a distinct physical meaning.

Considering that changes in pore volume are mainly determined by changes in bulk volume, it is essential to establish a link between the two quantities. This is done by eliminating $\dot{\sigma}$ from (2-3) to obtain the following expression for changes in pore volume per unit bulk volume:

$$\frac{\delta \dot{v}_p}{\delta v_b} = (1 - \alpha)\frac{\delta \dot{v}_b}{\delta v_b} + (1 - \alpha - \phi)C_m\dot{p} \tag{7}$$

It will be further assumed that deformations of the reservoir are uniaxial. This assumption is sufficiently accurate if the thickness of the reservoir is small compared to the depth below ground surface. In that case $\delta\dot{v}_b/\delta v_b$ is simply vertical strain rate in the reservoir.

In the subsequent work, flow in the reservoir of thickness h will be considered 2-dimensional and flow equations averaged along the reservoir thickness. If (7) is used in the left side of (1), the flow equation can be rewritten as follows:

$$-divq = \rho\phi\left[C_f + \frac{C_m}{\varphi}(1-\alpha-\phi)\right]\frac{\partial p}{\partial t} + \rho(1-\alpha)\frac{1}{h}\frac{\partial h}{\partial t} \tag{8}$$

where $C_f\rho = \partial\rho/\partial p$ is the fluid compressibility (written here for a single phase). For multiphase situation partial pressures should be used or C_f should be interpreted as a compressibility of the flowing mixture.

The last term in (8) is related to vertical strain rate in the reservoir. This quantity must be related to changes in vertical stress. This link can be established using the condition of no lateral strain in the reservoir and using isotropic elastic stress-strain law based on (3) but with shear deformations superimposed:

$$\dot{\varepsilon}_{ij} = \frac{1}{2G}(\dot{\sigma}_{ij} - \dot{\sigma}\delta_{ij}) + \frac{1}{3}(C_{b\sigma}\dot{\sigma} - C_{bp}\dot{p})\delta_{ij} \quad (\sigma = \sigma_{kk}/3) \tag{9}$$

where G is the shear modulus. The above relationship can be rewritten in the familiar form of generalized Hook's law if effective stress $\sigma' = \sigma - (1-\alpha)p\delta_{ij}$ is introduced.

During laterally constrained vertical deformations of the reservoir horizontal effective stress change becomes $\dot{\sigma}'_h = \nu_r/(1-\nu_r)\dot{\sigma}'_v$. Vertical strain rate $\dot{\varepsilon}_v$ can be calculated from (9) in terms of the vertical stress change as follows:

$$\dot{\varepsilon}_v = \frac{1}{3}C_r\frac{1+\nu_r}{1-\nu_r}[\dot{\sigma}_v - (1-\alpha)\dot{p}] = -\frac{1}{h}\frac{\partial h}{\partial t} \tag{10}$$

where ν_r is the Poisson's ratio and the left side above is the relative rate of reservoir thickness change. The last relationship will be used to relate pore pressure change in reservoir with total stress changes in overburden.

Reservoir-Overburden Interaction

Flow-related variation in pore pressure changes effective stress and results in deformations of both reservoir and the surrounding material. This, in turn, changes stresses in the reservoir and alters pore pressure as a results of deformation-related changes in the volume of pore space. The objective of this section is to determine a relationship between the reservoir pressure change and the vertical reservoir deformation, accounting for interaction between the reservoir and the surrounding material. Once the link between $\partial h/\partial t$ in (8) and pressure rate is established the flow equation (8) will be solved for a single injection/production well.

The problem of reservoir interaction with overburden will be solved assuming ideally elastic and isotropic overburden. The basis of the solution is the compatibility between deformations of the reservoir and of the surrounding material. From a mathematical point of view the reservoir will be considered as an infinitely thin deformable plane.

With the above assumptions the deformation filed in the overburden is continuous everywhere outside of the reservoir and is discontinuous across the reservoir plane. The situation is conceptually illustrated in Figure 1. The discontinuity in deformations across the reservoir develops

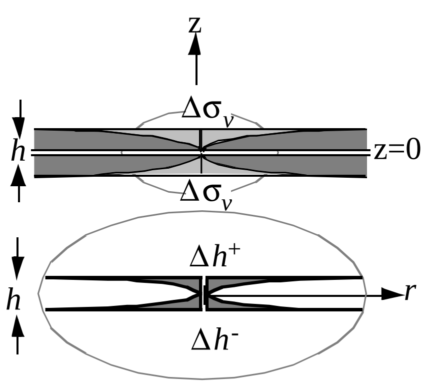

Figure 1: Representation of reservoir and definition of parameters.

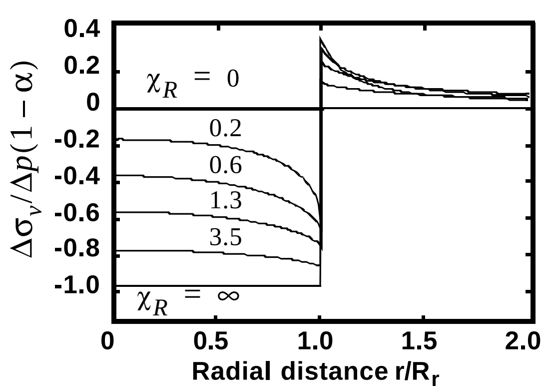

Figure 2: Total stress change due to pore pressure change in a circular reservoir

because the top of the reservoir moves down while the bottom moves up. Despite the discontinuity of deformations $\Delta h = \Delta h^+ - \Delta h^-$, vertical stress change $\Delta\sigma_v$ is continuous.

To relate Δh and $\Delta\sigma_v$ it is necessary to solve a problem of determining stress and deformation fields treating the displacement discontinuity as prescribed. This problem affords an analytical solution in the case when stress and deformation fields are axially-symmetric. Following Sneddon, 1951, equations of the theory of elasticity can be satisfied in an axially-symmetric case by introducing a potential $\Phi(r, z)$ such that all stress and displacement components are expressed in terms of $\Phi(r, z)$ derivatives. Stresses and displacements relevant to the current problem are expressed as follows:

$$u_z = \frac{2-2\nu_o}{1-2\nu_o}\nabla^2\Phi - \frac{1}{1-2\nu_o}\Phi_{zz}, \quad \sigma_z = E_o\frac{2-\nu_o}{1+\nu_o}\nabla^2\Phi_z - E_o\frac{1}{2(1+\nu_o)(1-2\nu_o)}\Phi_{zzz} \tag{11}$$

where E_o, ν_o are Young's modulus and Poisson's ratio of the overburden. The potential $\Phi(z, r)$ - must satisfy the biharmonic equation and the latter is solved using zero-order Hankel transform:

$$G(z, \xi) = \int_0^\infty \Phi(z, r) J_0(r\xi) r dr \quad , \quad \Phi(z, r) = \int_0^\infty G(z, \xi) J_0(\xi r) \xi d\xi$$

where $G(z, \xi)$ is the Hankel image of $\Phi(z, r)$.

The biharmonic equation for $\Phi(z, r)$ can be solved by applying Hankel transform with respect r to obtain an ordinary differential equation:

$$\left(\frac{d^2}{dz^2} - \xi^2\right)^2 G(z, \xi) = 0$$

Its solution is elementary and is as follows:

$$G(z, \xi) = (A + Bz)exp(z\xi) + (C + Dz)exp(-z\xi) \quad ,$$

where A, B, C, D are integration constants that can be chosen to satisfy a number of boundary conditions.

In this paper the reservoir is considered to be deep and it is not necessary to satisfy boundary condition on the free surface. Then, $z = 0$ can be taken as the reservoir plane, as in Figure 1. Since the influence of ground surface is neglected, the solution must be symmetrical around the reservoir plane and the solution for $z > 0$ can be considered only. Further, constants A, B must be zero, otherwise the solution will tend to infinity for large z. Also, since the reservoir plane is the plane of symmetry, shear stress at that plane must be zero. This leads to the relationship between the remaining constants: $\xi C = 2\nu_o D$, Sneddon, 1951. Finally, the solution can be given as follows:

$$G(z, \xi) = \frac{D}{\xi}(2\nu_o + z\xi)exp(-z\xi)$$

Substitution of the above solution into (11) gives the following expressions for stress and displacement in terms of unknown $D(\xi)$:

$$\Delta\sigma_v = -\frac{E_o}{(1+\nu_o)(1-2\nu_0)}\int_0^\infty \xi^3 D(\xi) J_0(r\xi)d\xi \quad , \quad \Delta h^+ = \frac{2-2\nu_o}{1-2\nu_o}\int_0^\infty \xi^2 D(\xi) J_0(r\xi)d\xi$$

Using notations $\Delta\bar{\sigma}(\xi)$ and $\Delta\bar{h}(\xi)$ for Hankel transforms of vertical stress and reservoir deformation, the above relationships give:

$$\Delta\bar{\sigma}_v(\xi) = \frac{E_o}{4(1-\nu^2)}\xi\Delta\bar{h}(\xi) \tag{12}$$

Note that Δh above is the full deformation across the whole reservoir, not Δh^+. This accounts for the coefficient 4.

The final step in reservoir-overburden analysis is to determine a link between pore pressure changes $\Delta p(r)$ and reservoir deformation $\Delta h(r)$. If Hankel transform is applied to (10), there will be the following linear relationship that involves the Hankel image of pore pressure:

$$-\frac{\Delta\bar{h}(\xi)}{h} = \frac{1}{3}C_r\frac{1+\nu_r}{1-\nu_r}[\Delta\bar{\sigma}_v(\xi) - (1-\alpha)\Delta\bar{p}(\xi)] \tag{13}$$

If $\Delta\bar{h}$ is eliminated from (12) and (13), the following set of expressions can be obtained:

$$\Delta\bar{\sigma}_v(\xi) = (1-\alpha)\frac{\chi\xi}{1+\chi\xi}\Delta\bar{p}(\xi) \quad , \quad \Delta\bar{\sigma}'_v(\xi) = (1-\alpha)\frac{1}{1+\chi\xi}\Delta\bar{p}(\xi) \tag{14}$$

$$\frac{\Delta\bar{h}(\xi)}{h} = \left(\frac{1}{3}C_r\frac{1+\nu_r}{1-\nu_r}\right)(1-\alpha)\frac{1}{1+\chi\xi}\Delta\bar{p}(\xi) \tag{15}$$

where:

$$\chi = \frac{C_r}{C_o}\frac{1+\nu_r}{1-\nu_r}\frac{1-2\nu_o}{1-\nu_o^2}\frac{h}{4}. \qquad (16)$$

Qualitative Features of Stress Redistribution

At this stage it is instructive to discuss solutions (15-16) in qualitative terms as well to give an example of using deceivingly simple relationships that involve Hankel images.

Most important features of the solution for stress change in overburden are controlled by a single constant, χ, defined by (16). This constant has a dimensionality of length and is of the order of reservoir thickness when stiffness properties of the reservoir and overburden are similar. In subsequent applications χ will enter various formulae in non-dimensional combinations of the type χ/l where l represents a characteristic dimension like a well or a reservoir radius, depending on the problem considered.

When the reservoir material is very stiff in relation to overburden or when its thickness is small in relation to the other characteristic dimension of the problem, $\chi/l \approx 0$, no stress redistribution occurs.

On the other hand, when the overburden is very stiff in relation to reservoir, or the reservoir thickness is large in comparison to a dimension like well radius, the overburden could constrain reservoir deformations to the point that there is no effective stress change in the reservoir. In this case of $\chi/l \gg 1$, $\Delta\sigma_v = (1-\alpha)\Delta p$, i.e. stresses induced in the overburden are directly controlled by local pore pressure change.

To understand the results in more quantitative form, consider an example of a pressure-depleted reservoir of thickness h and approximately represented by a circular area of radius R_r. Assume that the pressure drawdown is uniform across the reservoir and equal Δp for $r < R_r$ and zero outside of this area. For this pressure distribution the Hankel transform can be easily calculated: $\Delta p(\xi) = \Delta p R_r J_1(R_r\xi)/\xi$. Applying inverse Hankel transform to $\Delta\sigma_v(\xi)$ determined based on (15) gives the following expression for the total vertical stress change:

$$\frac{\Delta\sigma_v(r)}{\Delta p(1-\alpha)} = 1 - \int_0^\infty \frac{J_0(r\xi)J_1(c\xi)}{1+\chi\xi}d\xi$$

Figure 2 details vertical stress changes computed according to the above equation. The set of curves is for values of $\chi / R_r = \chi_r$ chosen is such that the corresponding stress values at $r = 0$ are evenly spaced. Vertical stress change at the center of the disk where pore pressure can be approximated as $\Delta\sigma_v(0)/\Delta p(1-\alpha) = \chi_r/(0.878+\chi_r)$ with three significant figures accuracy. Practical aspects of this solution will be discussed further.

In order to understand the physical meaning of solutions (15-16) it is instructive to convert relationships between Hankel transform into relationships between the characteristics of actual interest. If Hankel transform is applied to both parts of, say, equation (16) and $\Delta p(\xi)$ is expressed as a transform of $\Delta p(r)$, the following expression can be recovered:

$$\frac{\Delta h(r)}{h} = \left(\frac{1}{3}C_r\frac{1+\nu_r}{1-\nu_r}\right)(1-\alpha)\left(\Delta p(r) - \int_0^\infty K(r, r')\Delta p(r')r'dr'\right) \tag{17}$$

$$K(r, r') = \int_0^\infty \frac{\chi\xi}{1+\chi\xi}\xi J_0(r\xi)J_0(r'\xi)d\xi \tag{18}$$

The above relationship suggests that deformation at a point in the reservoir is related not only to pressure change at that particular point but is an integral effect of pressure changes in the rest of the reservoir. This “communication through overburden” depends on its stiffness in relation to the stiffness of the reservoir. When the parameter χ is zero, i.e. soft overburden, the conventional assumption of constant vertical stress in the reservoir is recovered from (18). In any realistic case, however, vertical stress is altered by pore pressure changes. The analytical form of (18) is such that the reservoir compression is always smaller when the stiffness of the overburden is accounted for ($\chi > 0$).

The kernel $K(r, r')$ of the integral expression (18) describes the influence of pressure changes at location r' on deformation at r.At $r = r'$ the kernel is singular and its direct utilization is difficult. These difficulties are avoided through the use of Hankel transform.

Flow problem

For a single phase flow with constant permeability the mass flux according to d’Arcy law can be taken as $q = -\rho(k/\mu)\partial p/\partial r$, where k is the absolute permeability and μ is the fluid viscosity. Substituting this expression into (1) and making use of (17)

$$\frac{1}{r}\frac{\partial}{r}\left(r\frac{k}{\mu}\frac{\partial p}{\partial r}\right) = \rho\phi c\left[\frac{\partial p}{\partial t} - \beta\int_0^\infty K(r, r')\frac{\partial p(r')}{\partial t}r'dr'\right] \quad , \tag{19}$$

where:

$$c = c_f + \frac{C_m}{\phi}(1-\alpha-\varphi) + \frac{C_r}{\phi}(1-\alpha)^2\frac{1-\upsilon_r}{3+3\nu_r} \;\; ; \;\; \beta = \frac{\frac{C_r}{\phi}(1-\alpha)^2\frac{1-\upsilon_r}{3+3\nu_r}}{c_f + \frac{C_m}{\phi}(1-\alpha-\varphi) + \frac{C_r}{\phi}(1-\alpha)^2\frac{1-\upsilon_r}{3+3\nu_r}} \tag{20}$$

The coefficient c in (20-21) is the combined compressibility of the fluid-reservoir system. It consists of three terms, the first one being the compressibility of the fluid, the second term is controlled by the compressibility of the solid rock matrix. This term describes the change in volume of the pore space due change in the volume of solid matrix resulting from variation in fluid pressure. The last term in c describes the change in the volume of pore space due to bulk reservoir compression caused by changes in effective stress and assuming that vertical stress does not change. In essence, c is the compressibility of the system when compaction drive is fully active, i.e. is not inhibited by the stiffness of the overburden.

The integral in the left-hand side of (19) describes inhibition of compaction due to vertical stress redistribution in the reservoir. When the parameters χ is large, corresponding to very stiff overburden in relation to the reservoir, the kernel $K(r, r')$ becomes δ-function and the entire integral is $\partial p(r)/\partial t$ so that the conventional well equation is recovered with the compressibility $c(1-\beta)$ i.e. lacking the component related to reservoir compaction.

Solution of Integro-Differential Equation

The governing equation of flow is solvable in closed form when permeability and density gradients are neglected. In this case the factor $\rho k/\mu$ is constant and application of Hankel transform to (20) gives the following ordinary differential equation for the Hankel image of pressure:

$$-\xi^2 p(\xi, t) = \frac{\mu\phi c}{k}\left(1-\beta\frac{\chi\xi}{1+\chi\xi}\right)\frac{dp(\xi, t)}{dt} \quad . \tag{21}$$

A family of solutions of (21) can be written as follows:

$$p(\xi, T) = Aexp\left(-\frac{\xi^2 T}{F(\xi)}\right) \quad F(\xi) = 1 - \beta\frac{\chi\xi}{1+\chi\xi} \quad T = \frac{kt}{\mu\phi c}. \tag{22}$$

where A is an arbitrary constant that can also depend on ξ.

An immediate interpretation of this analytical solution is difficult. When effects related to overburden-reservoir interaction are not present (e. g. when β or χ are zero), the above solution, transformed into physical space becomes as follows:

$$p(r, T) = \int_0^{\infty} Aexp(-\xi^2 T)\xi J_0(r\xi)d\xi = A\frac{1}{2T}exp\left(-\frac{r^2}{4T}\right) \tag{23}$$

Since integration of the above pressure over an infinite reservoir gives a time-independent constant, this solution corresponds to injection of a fixed mass of fluid at $T = 0$ into a reservoir with initially zero pressure. Assuming for the time being that he same interpretation holds true for the general case of (23), the solution corresponding to constant flow rate can be obtained by taking a function $p(\xi, T-\tau)$ corresponding to injection / extraction at $T = \tau$ and integrating it with respect to τ treating A as a constant. The fact that this procedure results in the solution corresponding to the constant rate of injection will be demonstrated directly after the solution is obtained. In a formal sense this procedure is legitimate since $p(\xi, T-\tau)$ is also a solution for an arbitrary τ. The same holds true for any integral with respect to τ. Integration with respect to τ gives the following solution:

$$\int_0^T p(\xi, T-\tau)d\tau = A(\xi)\frac{1}{\xi^2}\left(1 - \beta\frac{\chi\xi}{1+\chi\xi}\right)\left(1 - exp\left(\frac{\xi^2 T}{1 - \beta\frac{\chi\xi}{1+\chi\xi}}\right)\right) \tag{24}$$

The solution in physical space is obtained by applying Hankel transform to the pressure image (24) and selecting the constant $A(\xi)$ appropriately. The final result for pressure change in reservoir of thickness h due to flow rate q at the well is as follows:

$$\Delta p(r, T) = -\frac{q\mu}{2\pi kh}\int_0^{\infty}\left(1 - exp\left(-T\frac{\xi^2}{F(\xi)}\right)\right)J_0(r\xi)\frac{d\xi}{\xi} \quad ; \quad \left(F(\xi) = 1 - \beta\frac{\chi\xi}{1+\chi\xi}\right) \tag{25}$$

The unknown $A(\xi)$ was chosen to be proportional to $F^{-1}(\xi)$. The rational is the following. The final solution must be identical to the conventional line well solution in two limiting cases:

$\chi = 0$ and $\chi = \infty$. In the last case the solution must correspond to compressibility $c(1-\beta)$. With the mentioned choice of $A(\xi)$ both criteria are satisfied. In the first limiting case $F(\xi)$ in (25) is unity and the entire integral is the Hankel transform representation of the conventional solution since:

$$\int_0^\infty (1-exp(-T\xi^2))J_0(r\xi)\frac{d\xi}{\xi} = \frac{1}{2}Ei\left(-\frac{r^2}{4T}\right) ,$$

where $E_i(-x)$ is the exponential integral in terms of which the conventional solution is detailed.

When $\chi \to \infty$, $F(\xi) \to (1-\beta)$ and the time-related term in (25) becomes $T/(1-\beta)$, or $kt/\mu\phi c(1-\beta)$, i.e. it indeed corresponds to a conventional well solution with the compressibility $c(1-\beta)$.

Apart from this two limiting cases, the solution (25) is such that

$$\lim_{r \to 0} r(\partial \Delta p/\partial r) = -q\mu/2\pi kh$$

for all values of T, i.e. it corresponds to constant flow rate into the well at $r = 0$.

Qualitative features of the solution

The solution (25) can be best interpreted when pressure changes are detailed in terms of non-dimensional independent variables, $T_w = T/r_w^2$ and $R_w = r/r_w$, where r_w is the well radius introduced into the solution artificially since the base solution (26) corresponds to a line well. The introduction of r_w in this way preserves the form of the solution if parameters involved in (25) are replaced as follows:

$$r \to R_w = r/r_w,\ T \to T_w = T/r_w^2 = \frac{kt}{\mu\phi c r_w^2},\ \chi \to \chi_w = \chi/r_w = \frac{C_r}{C_o}\frac{1+\nu_r}{1-\nu_r}\frac{1-2\nu_o}{1-\nu_o^2}\frac{h}{4r_w}.$$

In the subsequent exposition the parameter χ_w will be referred to as the relative stiffness of reservoir-overburden system. Very soft overburden in relation to reservoir when no stress redistribution take place corresponds to $\chi_w = 0$ and the opposite case $\chi_w = \infty$ corresponds to a very stiff overburden When compressibilities of reservoir and overburden are the same and Poisson's ratios

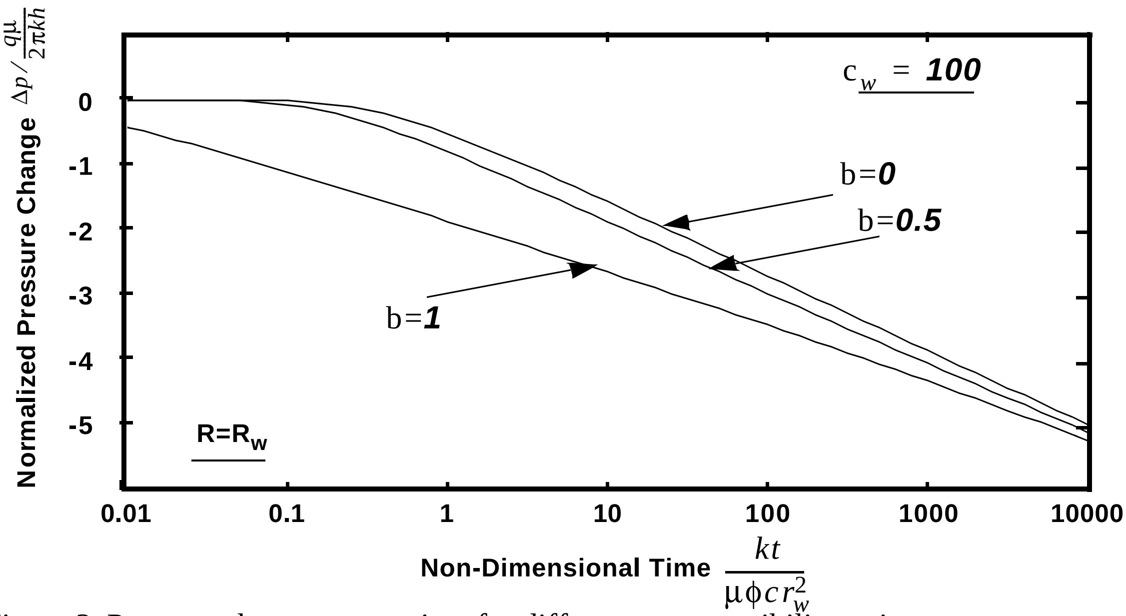

Figure 3: Pressure change versus time for different compressibility ratios

$\nu_r = \nu_o = 0.25$, $\chi_w = 2h/9r_w$.Typical practical values of relative stiffness in this case are 5 to 10, although much larger values are not uncommon.

Another parameter in (25), β, defined by (20) effectively characterizes compressibility of the reservoir rock matrix in relation to bulk compressibility of the reservoir-fluid system. When the fluid compressibility is low in relation to reservoir matrix compressibility, $\beta \approx 1$ and stress redistribution effects are the most pronounced. When fluid compressibility dominates the system, due to gas evolution for example, $\beta \approx 0$ and effects related to stress redistribution are negligeable.

Figure 3 illustrates flow pressure change (at the well) as a function of non-dimensional time. The case $\beta = 0$ corresponds to the conventional solution while in the limiting case $\beta = 1$ properties of the solution are entirely determined by stress redistribution effects. In all cases, however, pressure at the well drops faster compared to the conventional solution, as Figure 3 illustrates.

The reason for faster pressure drop compared to the conventional solution is related to vertical stress reduction that inhibits compaction (compared to the case when no stress redistribution take place). Figure 4 illustrates flow-induced vertical stress changes around the well at different times while Figure 5 depicts corresponding changes in fluid pressure.

The vertical stress reduction (Figure 4) in the vicinity of the well is always compensated by stress

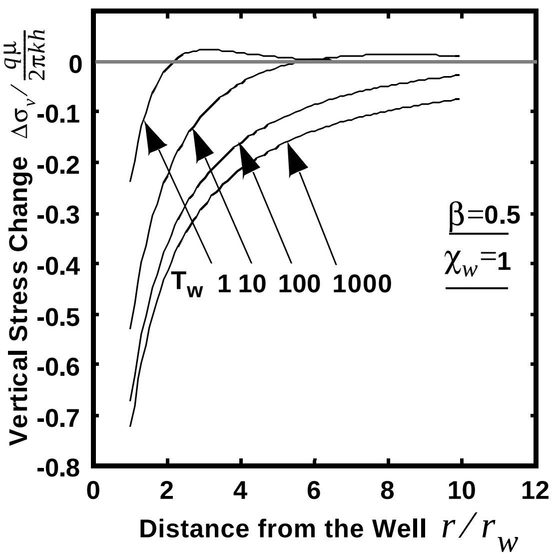

Figure 4: Vertical stress changes around the well at different times.

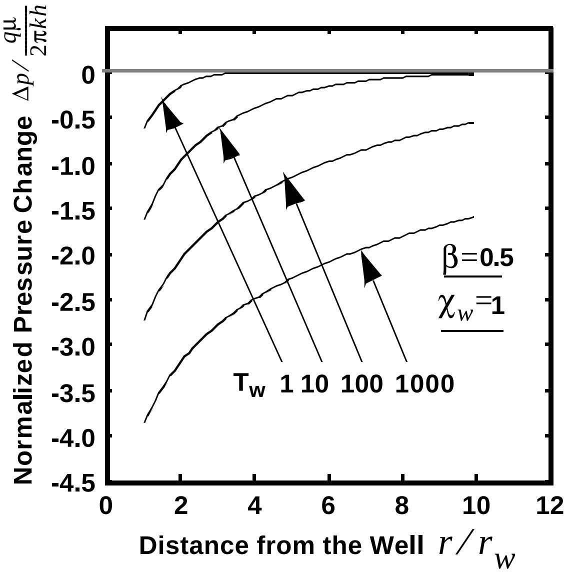

Figure 5: Fluid pressure changes around the well at different times.

increase elsewhere. However, the magnitude of stress increase is small since the load transferred from some area around the well is distributed over an infinite exterior of this area. Important qualitative effects related to stress redistribution are noticeable at early times when the area affected by pressure change is small and stress redistribution is reasonably localized. Figure 6 illustrates fluid pressure changes near a well at $T = 0.1$.

An interesting feature of pressure distributions detailed in Figure 6 is the presence a peak at some distance from the well. This peak is related to vertical stress increase as a result of load transfer from the near wellbore area. The insert in Figure 6 illustrates pressure changes at some distance from the well. It is quite clear that the initial increase in pressure is related to load transfer to areas not yet affected by flow. Fluid pressure start decreasing at some time when the flow front reaches the point of peak pressure.

The magnitude of pressure increase due to load transfer is not large since the load transferred from the area affected by flow is distributed over an infinite exterior of this area. Nevertheless,

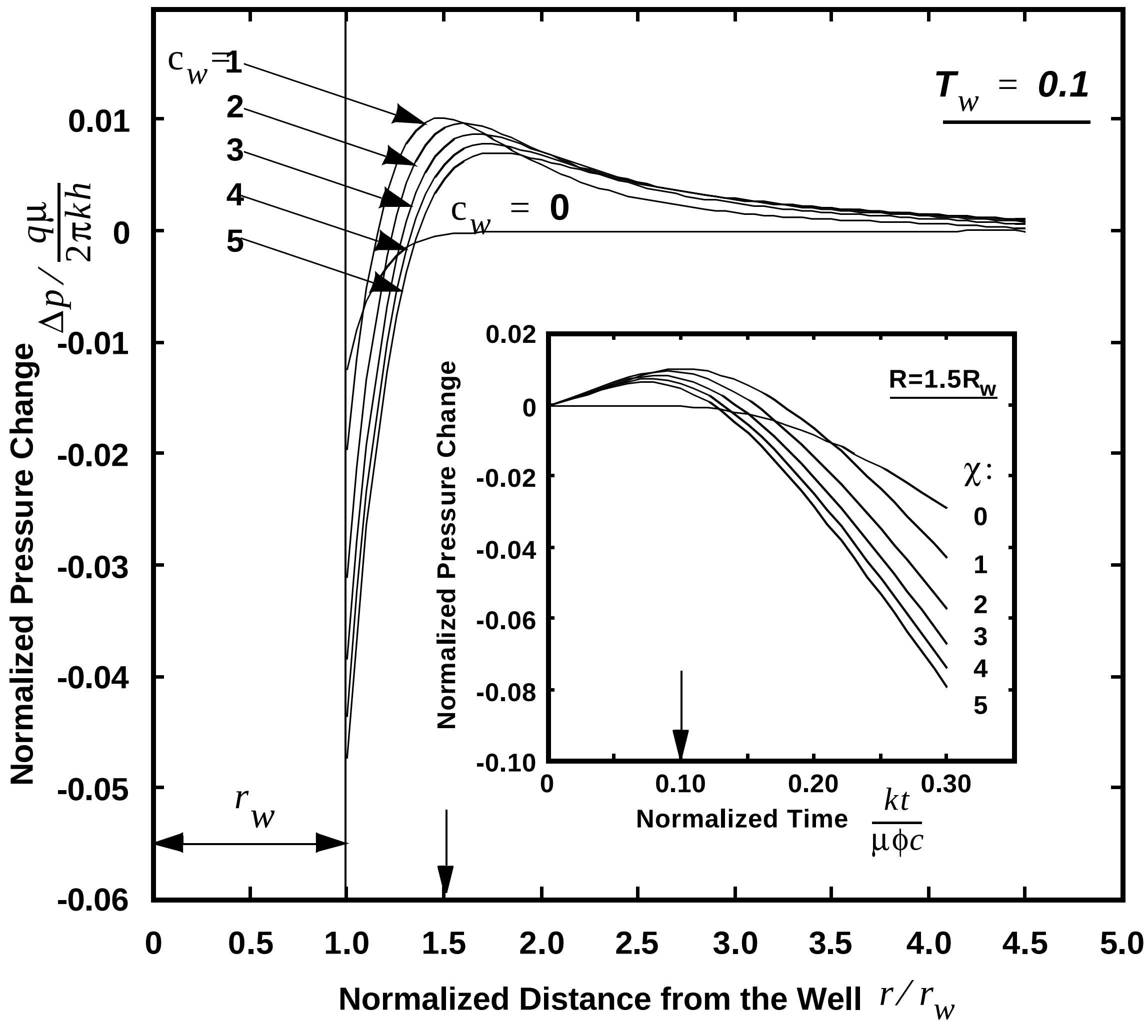

Figure 6: Normalized pressure change versus distance from the wellbore at early time for different relative stuffiness. Insert: Early time history of pressure change at a point.

since the entire load must be preserved, the accumulated effect of redistributed loads should be considerable.

Practical Implications - Single Well

Examination of the pattern of pressure changes around the wellbore (Figure 6) suggests that effects associated with flow-induced stress redistribution lead to sharper pressure gradients compared to the standard solution. Considering that only an immediate vicinity of the wellbore is affected, at least at early times, the phenomenon can be perceived as a skin effect. In order to appreciate the magnitude of this type of skin effect it is convenient to express the difference between the classical an the present solutions in terms of an equivalent skin factor. Defining the

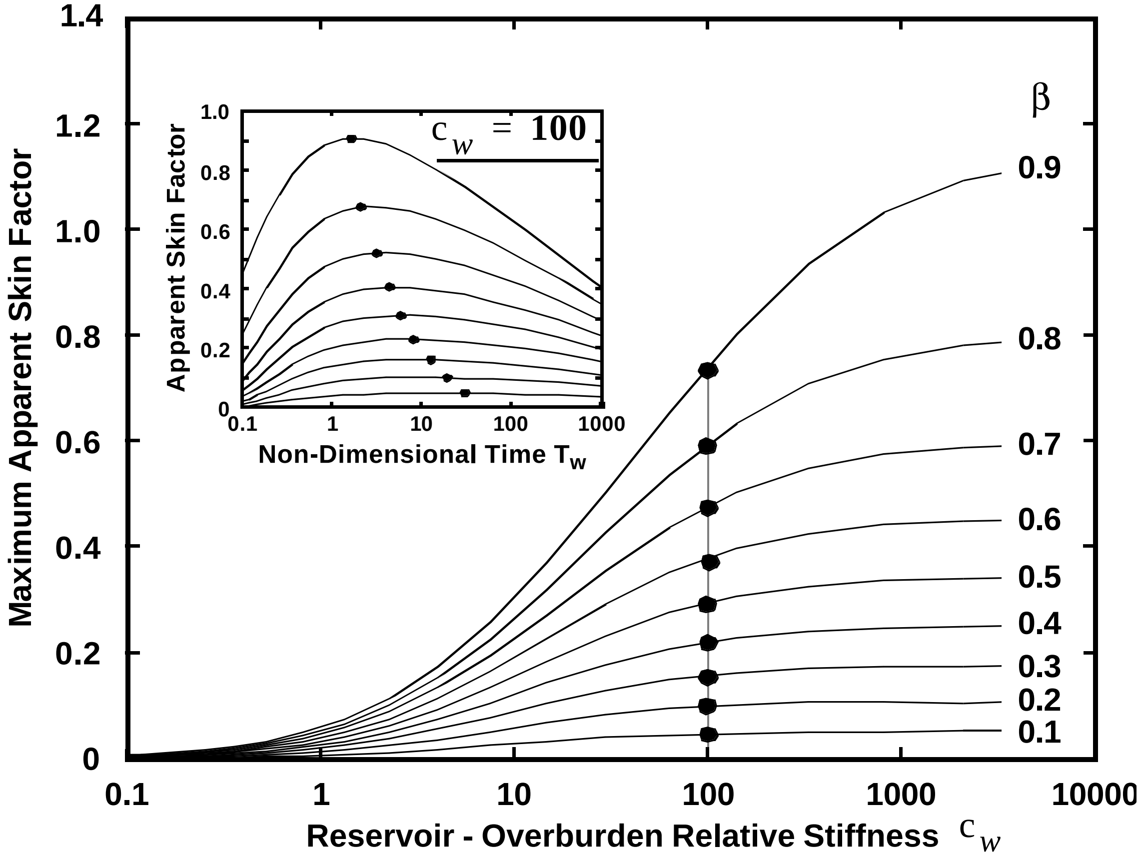

Figure 7: Maximum apparent skin factor versus reservoir-overburden relative stiffness. Insert: apparent skin factor versus time for different compressibility ratios.

skin factor s according to van Everdingen as $\Delta p_{skin} = s(q\mu/2\pi kh)$ and representing Δp according to (26) in terms of the classical solution corrected for skin effects, the skin factor can be determined from the following equation:

$$\Delta p = \frac{q\mu}{2\pi kh}\left(\frac{1}{2}E_i\left(-\frac{\phi\mu c r_w^2}{4kt}\right) - s\right)$$

The apparent skin factor determined in such a way is a function of time illustrated in the insert of Figure 7 where a set of curves for different β are shown At large times when effects of stress redistribution become insignificant, the skin factor tends to zero. Figure 7 illustrates the maximum apparent skin factor as a function of relative reservoir-overburden stiffness plotted for different values of relative compressibility. The range of skin factor values resulting from effects of stress redistribution suggests that this phenomenon in many cases can be perceived as an apparent well damage. The reduction in flow efficiency is neither dramatic nor negligeable.

The additional pressure reduction at the well is a direct consequence of inhibited compaction due to formation stiffness. The entire system behaves as if the compressibility of the rock matrix is lower compared to what is expected on the basis of a conventional analysis with full compressibility being active. For this reason the effects of inhibited compaction drive can be evaluated in terms of the compressibility reduction factor that can be applied to combined fluid-rock compressibility c defined by (20). The compressibility reduction factors F_c illustrated in Figure 8 are back-calculated in such a way as to match the conventional solution detailed for compressibility $c' = cF_c$ with the new solution (25). The equivalent compressibility c' determined in such a way is not a constant but a function of time. However, c' remains reasonably constant at early times and this type of interpretation of the new solution is legitimate, especially for cases of very compressible reservoirs when $\beta \approx 1$.

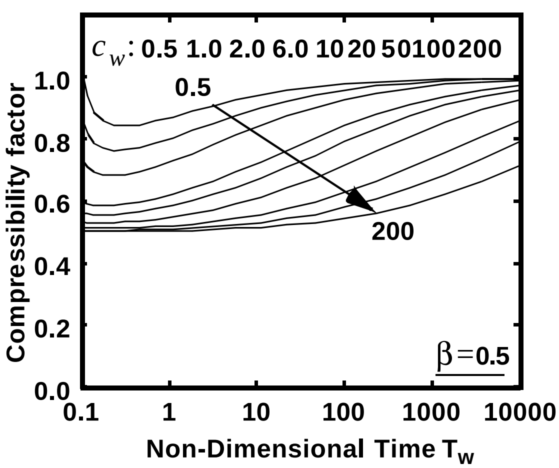

Figure 8: Compressibility reduction factors as a function of time ($\beta = 0.5$)

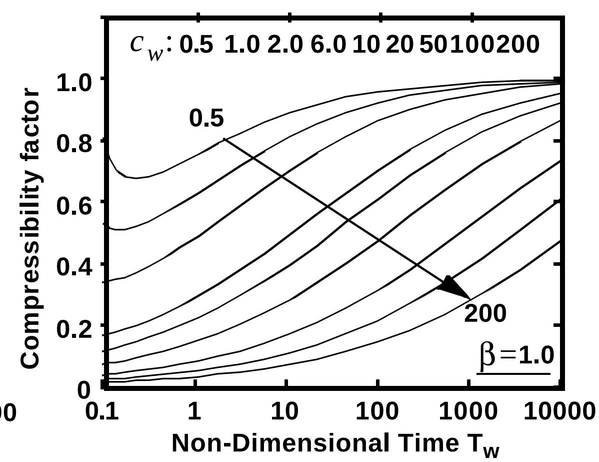

Figure 9: Compressibility reduction factors as a function of time ($\beta = 1.0$)

The fact that the compressibility factor in Figure 8 detailed for $\beta = 0.5$ is also 0.5 at early times effectively means that the compaction is completely inhibited, at least for relative compressibilities in excess of 200 and for non-dimensional flow times up to 100. This, perhaps, explains the reason why compaction was not observed in extended production tests at the Ekofisk North Sea reservoir. As a result of this early observation the possibility of compaction and the sea floor subsidence was dismissed leading to multimillion remedial measures when the production platform subsided some 10 ft., Sulak, 1991.

Practical Implications - Reservoir Scale

The developed solution is applicable only for flow into a single well in an infinite reservoir. In this case vertical stress is redistributed in such a way that the load never escapes the region beyond boundaries of the reservoir. For a finite reservoir, on the other hand, vertical stress reduction can affect the entire flow region. On a reservoir scale the degree of unloading will be strongly dependent on the ratio of the reservoir lateral dimension to its depth below ground surface. For shallow reservoirs of significant lateral extent the effect of unloading is expected to be negligeable. For deep reservoirs the unloading effect is estimated below.

Consider a circular reservoir of radius R_r where the fluid pressure uniformly dropped by Δp. Vertical stress changes as a result of pressure depletion were already examined, Figure 2. The degree of unloading in this case is controlled by a relative stiffness χ_R defined as follows:

$$\chi_R = \frac{C_r}{C_o}\frac{1+\nu_r}{1-\nu_r}\frac{1-2\nu_o}{1-\nu_o^2}\frac{h}{4R_r}$$

Changes in effective stress (leading to compaction) are strongly related to χ_R and illustrated in Figure 10.

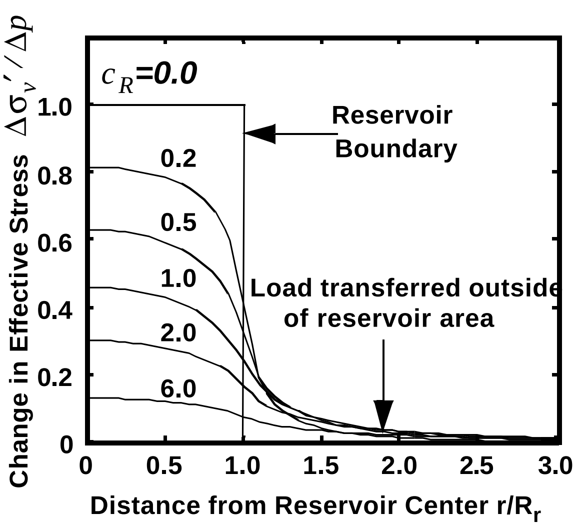

Figure 10: Changes in effective stress as a result of reservoir depletion

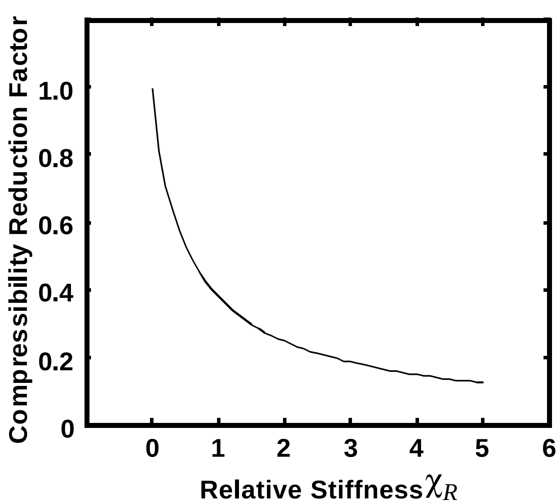

Figure 11: Apparent compressibility reduction factor

Considering that changes in the effective stress in the reservoir region are lower compared to the case when load transfer does not take place, the overall reservoir compaction is also lower. This affects the amount of oil recovered as a result of the compaction drive. Figure 11 illustrates the compressibility reduction factor appropriate for assessment of effective compressibility when stress redistribution takes place. This assessment suggests that estimates of compaction-related hydrocarbon recovery can be substantially in error if the load transfer away from the reservoir region is not accounted for.

Conclusions

The phenomenon of flow-related stress changes around a well was investigated by deriving and solving a modified diffusivity equation in which the storage term related to reservoir compaction is determined by changes in vertical stress resulting from interaction between reservoir and overburden. A link between the reservoir pressure changes and changes in vertical stress is obtained by solving the theory of elasticity problem for stresses and displacements in material surrounding the reservoir and assuming continuity of displacements at the interface between the reservoir and the host material.

The equation governing radial flow into a line well is an integro-differential equation in which the term related to reservoir compaction represents an integral effect of pressure changes everywhere in the reservoir. An analytical solution of the coupled stress - flow problem became possible because both stress interaction and flow problems are solvable using the same mathematical technique based on Hankel transform of governing equations.

Detailed examination of the solution suggests that effects related to stress redistribution are far from negligible, especially when the compressibility of the reservoir matrix exceeds that of a fluid. When pressure in the reservoir drops and the reservoir material tends to compact, tensile deformations are induced in the overburden that reacts to reduce the reservoir compaction. The effectiveness of this reaction to prevent reservoir deformation is dependent of the relative stiffness of the reservoir in relation to stiffness of the overburden. When overburden is “soft” relative to reservoir, the constraining effect is negligeable.

The mechanism of stress-flow interaction is such that changes in pressure distribution are the most pronounced in the vicinity of the well. The reservoir - overburden stress interaction leads to sharper pressures gradients near the well and more rapid initial pressure decline compared to the standard case. If well test data are interpreted in conventional terms, effects associated with stress redistribution will be perceived as a skin effect and accounted for within the standard interpretation scheme by a skin factor. Estimates based on the obtained solution indicate that stress redistribution effects in most cases would amount to skin of up to +1 and possibly higher.

An alternative way of assessing the constraining effect of overburden is in terms of an equivalent compressibility that can be used within the standard well test interpretation scheme. Estimates presented in the paper have shown that compressibility reduction factors that account for reservoir-overburden interaction are strongly dependent on reservoir compressibility in relation to fluid compressibility as well as on the relative stiffness of overburden in relation to reservoir stiffness. When bulk moduli of reservoir and overburden are the same, the reservoir - overburden interaction leads to almost complete inhibition of compaction drive at early flow times. Eventually, the benefits of the compaction drive are fully restored. This result suggests that in cases where reservoir compaction is a significant factor, conventional reservoir simulation schemes will give a distorted view of an early economics of the project.

Assessment of the effects of stress redistribution at a reservoir scale suggests a potential for considerable errors in estimates of recoverable reserves if the described effects are not properly accounted for. Specific estimates require information on reservoir size in relation to depth below ground surface, terminal drawdown and material stuffiness. The present solution has been obtained for reservoirs at infinite depth and the compressibility reduction factor in Figure 11 is only appropriate for deep reservoirs of limited extent.

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